No. 192 August 2021

Second-Order Productivity, Second-Order Payoffs, and the Owen Value

André Casajus[a,b] and Rodrigue Tido Takeng[c]

[a] *Prof. Dr. André Casajus is Research Professor at the Chair of Ecomics and Information Systems at HHL Leipzig Graduate School of Management, Leipzig, Germany. Email: andre.casajus@hhl.de*

[b] *Bartender, Dr. Hops Craft Beer Bar, Eichendorffstr. 7, Leipzig, Germany*

[c] *Rodrigue Tido Takeng is a research associate at the Centre de Recherche en Economie et Management (CREM) at Normandie University (UNICAEN), France.*

We introduce the concepts of the components' second-order productivities in cooperative games with transferable utility (TU games) with a coalition structure (CS games) and of the components' second-order payoffs for one-point solutions for CS games as generalizations of the players' second-order productivities in TU games and of the players' second-order payoffs for one-point solutions for TU games (Casajus, 2021, Discrete Appl. Math. 304, 212-219). The players' second-order productivities are conceptualized as second-order marginal contributions, that is, how one player affects another player's marginal contributions to coalitions containing neither of them by entering these coalitions. The players' second-order payoffs are conceptualized as the effect of one player leaving the game on the payoff of another player. Analogously, the components' second-order productivities are conceptualized as their second-order productivities in the game between components; the components' second-order payoffs are conceptualized as their second-order payoffs in the game between components. We show that the Owen value is the unique efficient one-point solution for CS games that reflects the players' and the components' second-order productivities in terms of their second-order payoffs.

Second-order productivity, second-order payoffs, and the Owen value

André Casajus[a,b,*], Rodrigue Tido Takeng[c]

[a] *HHL Leipzig Graduate School of Management, Jahnallee 59, 04109 Leipzig, Germany*
[b] *Dr. Hops Craft Beer Bar, Eichendorffstr. 7, 04277 Leipzig, Germany*
[c] *CREM (CNRS 6211), Normandie University, UNICAEN, France. TEPP (CNRS 2042)*

Abstract

We introduce the concepts of the components' second-order productivities in cooperative games with transferable utility (TU games) with a coalition structure (CS games) and of the components' second-order payoffs for one-point solutions for CS games as generalizations of the players' second-order productivities in TU games and of the players' second-order payoffs for one-point solutions for TU games (Casajus, 2021, Discrete Appl. Math. 304, 212–219). The players' second-order productivities are conceptualized as second-order marginal contributions, that is, how one player affects another player's marginal contributions to coalitions containing neither of them by entering these coalitions. The players' second-order payoffs are conceptualized as the effect of one player leaving the game on the payoff of another player. Analogously, the components' second-order productivities are conceptualized as their second-order productivities in the game between components; the components' second-order payoffs are conceptualized as their second-order payoffs in the game between components. We show that the Owen value is the unique efficient one-point solution for CS games that reflects the players' and the components' second-order productivities in terms of their second-order payoffs.

Keywords: TU game, Shapley value, Owen value, second-order marginal contributions, second-order payoffs
2010 MSC: 91A12 *JEL:* C71, D60

1. Introduction

A cooperative game with transferable utility for a finite player set (TU game or simply game) is given by a coalition function that assigns a worth to any coalition (subset of the player set), where the empty coalition obtains zero. (One-point) solutions for TU games

*corresponding author
Email addresses: mail@casajus.de (André Casajus), rodriguetido@yahoo.fr (Rodrigue Tido Takeng)
URL: www.hhl.de/casajus (André Casajus)

assign a payoff to any player in any TU game. The Shapley value (Shapley, 1953) probably is the most eminent one-point solution concept for TU games. And its characterization by Young (1985) probably is its most important one.

Young (1985) characterizes the Shapley value by three properties of solutions: efficiency, symmetry, and marginality or strong monotonicity. Efficiency: the players' payoffs sum up to the worth generated by the grand coalition. Symmetry: equally productive[1] players obtain the same payoff. Marginality: a player's payoff only depends on her own productivity. Strong monotonicity: whenever a player's productivity in a game weakly increases so does her payoff. Note that strong monotonicity implies marginality. This result indicates that the Shapley value is *the* efficient solution that reflects the players' productivities by their payoffs.

The organization of players into groups can be modelled by coalition structures—partitions of the player set. Games enriched with a coalition structure are addressed as CS games and the corresponding solutions as CS solutions. Owen (1977) generalizes of the Shapley value into an efficient CS solution where the components of the coalition structure are treated like players.[2] Khmelnitskaya and Yanovskaya (2007) provide a characterization of the Owen value that breathes the spirit of Young's (1985) characterization of the Shapley value.[3] This characterization uses four properties: efficiency, marginality, symmetry within components, and symmetry across components. Symmetry across components: components that are equally productive in the game between components obtain the same sum of payoffs of their members.

Recently, Casajus (2021) suggests a second-order version of Young's (1985) characterization of the Shapley value. This characterization is based on the notions of the players' second-order productivities and second-order payoffs. A player's second-order productivity with respect to another player reflects how the former affects the latter player's marginal contribution to coalitions containing neither of them by entering these coalitions; a player's second-order payoff with respect to another player reflects how the former affects the latter player's payoff by leaving the game. The Shapley value is the unique efficient solution the reflects the players' second-order productivities in terms of their second-order payoffs. More precisely, it is the unique solution that satisfies efficiency and second-order versions of symmetry and marginality. Second-order symmetry: players who are equally second-order productive with respect to a third player obtain the same second-order payoff with respect to this third player. Second-order marginality: a player's second-order payoff with respect to another player only depends on her own second-order productivity with respect to this other player.

[1] In this paper, a player's productivity in a game refers to her influence on the generation of worth as expressed by her marginal contributions to coalitions not containing her, that is, the differences between the worth generated after she entered such a coalition and the worth generated before she entered.

[2] Alternative efficient CS solutions have been suggested by Kamijo (2009) and Alonso-Meijide et al. (2014), for example. Alternative non-efficient CS solutions have been suggested by Aumann and Drèze (1974), Owen (1982), and Alonso-Meijide and Fiestras-Janeiro (2002), for example.

[3] Alternative characterizations of the Owen value have been suggested by Owen (1977) himself, Hart and Kurz (1983), Calvo et al. (1996), Hamiache (2001), Albizuri (2008), and Casajus (2010), for example.

In this paper, we suggest a second-order version of Khmelnitskaya and Yanovskaya's (2007) characterization of the Owen value. In particular, we show that the Owen value is the unique CS solution that satisfies efficiency, second-order marginality, and second-order versions of symmetry within components and symmetry across components. (Theorem 10). Second-order symmetry within components is just the restriction of second-order symmetry to players within the same component. Second-order symmetry across components: components that are equally second-order productive in the game between components obtain the same sum of second-order payoffs of their members. This result is partly based on three facts. Second-order marginality implies marginality (Proposition 6). Efficiency and second-order symmetry within components imply symmetry within components (Proposition 7). Efficiency and second-order symmetry across components imply symmetry across components (Proposition 9).

The remainder of this paper is organized as follows. In Section 2, we provide basic definitions and notation. In Section 3, we survey the characterizations of the Shapley value by Young (1985) and by Casajus (2021). In Section 4, we first survey the characterization of the Owen value by Khmelnitskaya and Yanovskaya (2007). Then, we provide our second-order approach to the Owen value. Some remarks conclude the paper.

2. Basic definitions and notation

Let the universe of players $\mathfrak{U}$ be a countably infinite set, and let $\mathcal{N}$ denote the set of all finite subsets of $\mathfrak{U}$. The cardinalities of $S, T, N \in \mathcal{N}$ are denoted by s, t, and n, respectively. A (finite TU) game for the player set $N \in \mathcal{N}$ is given by a **coalition function** $v : 2^N \to \mathbb{R}$, $v(\emptyset) = 0$, where 2^N denotes the power set of N. Subsets of N are called **coalitions**; $v(S)$ is called the worth of coalition S. The set of all games for N is denoted by $\mathbb{V}(N)$; the set of all games is denoted by $\mathbb{V} := \bigcup_{N \in \mathcal{N}} \mathbb{V}(N)$.

For $N \in \mathcal{N}$, $T \subseteq N$, and $v \in \mathbb{V}(N)$, the **subgame** $v|_T \in \mathbb{V}(T)$ is given by $v|_T(S) = v(S)$ for all $S \subseteq T$; for $i \in N$ and $S \subseteq N$, we occasionally write v_{-i} and v_{-S} instead of $v|_{N \setminus \{i\}}$ and $v|_{N \setminus S}$, respectively. For $N \in \mathcal{N}$, $v, w \in \mathbb{V}(N)$, and $\alpha \in \mathbb{R}$, the coalition functions $v + w \in \mathbb{V}(N)$ and $\alpha \cdot v \in \mathbb{V}(N)$ are given by $(v + w)(S) = v(S) + w(S)$ and $(\alpha \cdot v)(S) = \alpha \cdot v(S)$ for all $S \subseteq N$. For $T \subseteq N$, $T \neq \emptyset$, the game $u_T^N \in \mathbb{V}$ given by $u_T^N(S) = 1$ if $T \subseteq S$ and $u_T^N(S) = 0$ otherwise is called a **unanimity game**. Any $v \in \mathbb{V}(N)$, $N \in \mathcal{N}$ can be uniquely represented by unanimity games. In particular, we have

$$v = \sum_{T \subseteq N : T \neq \emptyset} \lambda_T(v) \cdot u_T^N, \tag{1}$$

where the coefficients $\lambda_T(v)$ are known as the Harsanyi dividends (Harsanyi, 1959) and can be determined recursively by

$$\lambda_T(v) := v(T) - \sum_{S \subsetneq T : S \neq \emptyset} \lambda_S(v). \tag{2}$$

Players $i, j \in N$ are called **symmetric** in $v \in \mathbb{V}(N)$ if $v(S \cup \{i\}) - v(S) = v(S \cup \{j\}) - v(S)$ for all $S \subseteq N \setminus \{i, j\}$.

A **rank order** of $N \in \mathcal{N}$ is a bijection $\rho : N \to \{1, 2, \ldots, |N|\}$ with the interpretation that i is the $\rho(i)$th player in ρ; the set of rank orders of N is denoted by $R(N)$. The set of players before i in ρ is denoted by $B_i(\rho) := \{\ell \in N : \rho(\ell) < \rho(i)\}$. The **marginal contribution** of i in ρ and $v \in \mathbb{V}(N)$ is denoted by

$$MC_i^v(\rho) := v(B_i(\rho) \cup \{i\}) - v(B_i(\rho)). \tag{3}$$

A **solution** for $\mathbb{V}$ is an operator that assigns to any $N \in \mathcal{N}$, $v \in \mathbb{V}(N)$, and $i \in N$ a payoff $\varphi_i(v)$. The **Shapley value** (Shapley, 1953) for $\mathbb{V}$, Sh, is given by

$$\mathrm{Sh}_i(v) := \sum_{T \subseteq N : i \in T} \frac{\lambda_T(v)}{t} = \sum_{S \subseteq N \setminus \{i\}} \frac{v(S \cup \{i\}) - v(S)}{n \cdot \binom{n-1}{s}} = \sum_{\rho \in R} \frac{1}{|R(N)|} \cdot MC_i^v(\rho) \tag{4}$$

for all $N \in \mathcal{N}$, $v \in \mathbb{V}(N)$, and $i \in N$.

For $N \in \mathcal{N}$, let $\mathfrak{P}(N)$ denote the set of all partitions (**coalition structures**) of N; the component of player $i \in N$ in $\mathcal{P} \in \mathfrak{P}(N)$ is denoted by $\mathcal{P}(i)$. For $N \in \mathcal{N}$, $\mathcal{P} \in \mathfrak{P}(N)$, $T \subseteq N$ and $i \in N$, let $\mathcal{P}(T) \subseteq \mathcal{P}$ be given by $\mathcal{P}(T) := \{P \in \mathcal{P} \mid T \cap P \neq \emptyset\}$, let $\mathcal{P}|_T \in \mathfrak{P}(T)$ be given by $\mathcal{P}|_T := \{T \cap P \mid P \in \mathcal{P}(T)\}$, let $\mathcal{P}_{-T} \in \mathfrak{P}(N \setminus T)$ be given by $\mathcal{P}_{-T} := \mathcal{P}|_{N \setminus T}$, and let $\mathcal{P}_{-i} \in \mathfrak{P}(N \setminus \{i\})$ be given by $\mathcal{P}_{-i} := \mathcal{P}_{-\{i\}}$.

A **CS game** for $N \in \mathcal{N}$ is a pair $(v, \mathcal{P})$, where $v \in \mathbb{V}(N)$ and $\mathcal{P} \in \mathfrak{P}(N)$. Let $\mathbb{VP}(N)$ denote the set of all CS games for N and let $\mathbb{VP} := \cup_{N \in \mathcal{N}} \mathbb{VP}(N)$ denote the set of all CS games.

A **(CS) solution** for $\mathbb{VP}$ is an operator φ that assigns to any $N \in \mathcal{N}$, $i \in N$, and $(v, \mathcal{P}) \in \mathbb{VP}(N)$ a payoff $\varphi_i(v, \mathcal{P})$; for $P \in \mathcal{P}$, we set $\varphi_P(v, \mathcal{P}) = \sum_{i \in P} \varphi_i(v, \mathcal{P})$. For $N \in \mathcal{N}$ and $\mathcal{P} \in \mathfrak{P}(N)$, the set of all rank orders that respect $\mathcal{P}$ is denoted by

$$R(N, \mathcal{P}) := \{\rho \in R(N) \mid \text{for all } P \in \mathcal{P} \text{ and } i, j \in P : |\rho(i) - \rho(j)| < |P|\},$$

that is, in any such rank order, the players from any component follow each other without players from other components between them. The **Owen value** (Owen, 1977) for $\mathbb{VP}$, Ow, is the CS solution given by

$$\mathrm{Ow}_i(v, \mathcal{P}) := \sum_{T \subseteq N : i \in T} \frac{\lambda_T(v)}{|\mathcal{P}(i) \cap T| \cdot |\mathcal{P}(T)|} \tag{5a}$$

$$= \sum_{\mathcal{C} \subseteq \mathcal{P} \setminus \{\mathcal{P}(i)\}} \sum_{S \subseteq \mathcal{P}(i) \setminus \{i\}} \frac{v\left(S \cup \{i\} \cup \bigcup_{C \in \mathcal{C}} C\right) - v\left(S \cup \bigcup_{C \in \mathcal{C}} C\right)}{|\mathcal{P}(i)| \cdot \binom{|\mathcal{P}(i)|-1}{s} \cdot |\mathcal{P}| \cdot \binom{|\mathcal{P}| - 1}{|\mathcal{C}|}} \tag{5b}$$

$$= \sum_{\rho \in R(N, \mathcal{P})} \frac{1}{|R|} \cdot MC_i^v(\rho) \tag{5c}$$

for all $N \in \mathcal{N}$, $i \in N$, and $(v, \mathcal{P}) \in \mathbb{VP}(N)$.

Fix an injection $\imath : \mathcal{N} \to \mathfrak{U}$, $N \mapsto \imath_N$ for $N \in \mathcal{N}$. For any $N \in \mathcal{N}$, $\mathcal{P} \in \mathcal{P}(N)$, and $v \in \mathbb{V}(N)$, set $[\mathcal{P}] := \{\imath_P \mid P \in \mathcal{P}\}$ and let $v_{\mathcal{P}} \in \mathbb{V}([\mathcal{P}])$ be given by

$$v_{\mathcal{P}}([\mathcal{C}]) := v\left(\bigcup_{C \in \mathcal{C}} C\right) \qquad \text{for all } \mathcal{C} \subseteq \mathcal{P}. \tag{6}$$

The TU game $v_{\mathcal{P}}$ is called the **game between components** or **intermediate game** for the CS game $(v, \mathcal{P})$. For $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, and $P \in \mathcal{P}$, we have

$$\mathrm{Ow}_P(v, \mathcal{P}) = \mathrm{Sh}_{\imath_P}(v_{\mathcal{P}}). \tag{7}$$

3. The Shapley value[4]

The marginal contributions of a player $i \in N$, $N \in \mathcal{N}$ in the game $v \in \mathbb{V}(N)$ given as

$$v(S \cup \{i\}) - v(S), \qquad S \subseteq N \setminus \{i\} \tag{8}$$

indicate her (individual) productivity or contribution to the generation of worth in the game v. The right-hand formula of the Shapley value in (4) indicates that the players' Shapley value payoffs reflects their productivities in games as expressed by their own marginal contributions. Young (1985) shows that the Shapley value is the unique efficient such solution.

Efficiency, E. For all $N \in \mathcal{N}$ and $v \in \mathbb{V}(N)$, we have $\sum_{\ell \in N} \varphi_\ell(v) = v(N)$.

Symmetry, S. For all $N \in \mathcal{N}$, $v \in \mathbb{V}(N)$, and $i, j \in N$ such that i and j are symmetric in v, we have $\varphi_i(v) = \varphi_j(v)$.

Marginality, M. For all $N \in \mathcal{N}$, $v, w \in \mathbb{V}(N)$, and $i \in N$ such that $v(S \cup \{i\}) - v(S) = w(S \cup \{i\}) - w(S)$ for all $S \subseteq N \setminus \{i\}$, we have $\varphi_i(v) = \varphi_i(w)$.

Theorem 1 (Young, 1985). *The Shapley value is the unique solution for $\mathbb{V}$ that satisfies efficiency (**E**), symmetry (**S**), and marginality (**M**).*[5]

Symmetry and marginality can be paraphrased as follows. Symmetry: players who are equally productive in a game should obtain the same payoff. Marginality: a player who is equally productive in two games should obtain the same payoff in these games. Therefore, a solution that is intended to reflect the players' productivities should satisfy these properties.

Later on, Casajus (2021) introduces the notions of the players' second-order productivities and second-order payoffs. Second-order productivities are conceptualized as second-order marginal contributions: the second-order marginal contributions of player $i \in N$, $N \in \mathcal{N}$ with respect to player $j \in N \setminus \{i\}$ in a game $v \in \mathbb{V}(N)$ are given as

$$[v(S \cup \{i, j\}) - v(S \cup \{i\})] - [v(S \cup \{j\}) - v(S)], \qquad S \subseteq N \setminus \{i, j\}. \tag{9}$$

[4]This section partly follows Casajus (2021).

[5]Originally, Young (1985) invokes anonymity (called symmetry by him) instead of symmetry (in our parlance). Although anonymity is stronger than symmetry, it is well-known and easy to check that anonymity can be replaced with symmetry in his characterization. Moreover, his characterization works on fixed player sets.

These describe how player i affects the productivity of player j.[6] The second-order payoff of player $i \in N$, $N \in \mathcal{N}$ with respect to player $j \in N \setminus \{i\}$ in a game $v \in \mathbb{V}(N)$ is given by

$$\varphi_j(v) - \varphi_j(v_{-i}).$$

It describes how player i affects the payoff of player j.[7]

Based on these notions, Casajus (2021) motivates natural second-order versions of symmetry and marginality. For all $N \in \mathcal{N}$, $v \in \mathbb{V}(N)$, and $i, j, k \in N$, $i \neq j \neq k \neq j$, players i and j are called **second-order symmetric with respect to player** k if

$$\begin{aligned}&[v(T \cup \{i,k\}) - v(T \cup \{i\})] - [v(T \cup \{k\}) - v(T)] \\ &\quad = [v(T \cup \{j,k\}) - v(T \cup \{j\})] - [v(T \cup \{k\}) - v(T)]\end{aligned}$$

for all $T \subseteq N \setminus \{i,j,k\}$.

Second-order symmetry, 2S. For all $N \in \mathcal{N}$, $v \in \mathbb{V}(N)$ and $i, j, k \in N$, $i \neq j \neq k \neq j$ such that players i and j are second-order symmetric with respect to player k, we have

$$\varphi_k(v) - \varphi_k(v_{-i}) = \varphi_k(v) - \varphi_k(v_{-j}).$$

Second-order marginality, 2M. For all $N \in \mathcal{N}$, $v, w \in \mathbb{V}(N)$ and $i, j \in N$, $i \neq j$ such that

$$\begin{aligned}&[v(T \cup \{i,j\}) - v(T \cup \{i\})] - [v(T \cup \{j\}) - v(T)] \\ &\quad = [w(T \cup \{i,j\}) - w(T \cup \{i\})] - [w(T \cup \{j\}) - w(T)]\end{aligned}$$

for all $T \subseteq N \setminus \{i,j\}$, we have

$$\varphi_j(v) - \varphi_j(v_{-i}) = \varphi_j(w) - \varphi_j(w_{-i}).$$

Second-order symmetry and second-order marginality can be paraphrased as follows. Second-order symmetry: players who are equally second-order productive with respect to a third player in a game should be assigned the same second-order payoff with respect to the latter. Second-order marginality: a player who is equally second-productive with respect to a another player in two games should be assigned the same second-order payoff with respect to the latter in these games. Therefore, it seems to be plausible that a solution the second-order payoffs of which are intended to reflect the players' second-order productivities satisfies these properties.

It turns out that the Shapley value reflects the players' second-order productivities in terms of their second-order payoffs in the same vein as it reflects the players' (first-order) productivities in terms of their (first-order) payoffs.

[6] The second-order marginal contributions of player i to player j in the game v equal player j's contributions to player i. Often, these are referred to as the second-order derivative of v with respect to i and j.

[7] Second-order (and higher-order) Casajus and Huettner (2018, Definition 9) introduce second-order (and higher-order) payoffs as second-order (and higher-order) contributions.

Theorem 2 (Casajus, 2021). *The Shapley value is the unique solution for* $\mathbb{V}$ *that satisfies efficiency (**E**), second-order symmetry (**2S**), and second-order marginality (**2M**).*

The proof of this theorem uses the fact that second-order marginality implies marginality, the proof of which is rather short.

Proposition 3 (Casajus, 2021). *If a solution for* $\mathbb{V}$ *satisfies second-order marginality (**2M**), then it satisfies marginality (**M**).*

Nevertheless, the proof of Theorem 2 is much more involved than the proof of Theorem 1 due to use of second-order symmetry instead of symmetry.

On the one hand, second-order symmetry does not inply symmetry (Casajus, 2021, Remark 3). On the other hand, the counterexamples in Casajus (2021, Remark 3) fail efficiency. As our first result, we show that the proof of Theorem 2 can be simplified substantially by providing a rather short proof that efficiency and second-order symmetry imply symmetry.

Proposition 4. *If a solution for* $\mathbb{V}$ *satisfies second-order symmetry (**2S**) and efficiency (**E**), then it satisfies symmetry (**S**).*

Proof. Let the solution φ satisfy **2S** and **E**. For $|N| = 1$, nothing is to show. Let now $|N| > 1$. Let (*) $i, j \in N$, $i \neq j$, $N \in \mathcal{N}$ be symmetric in $v \in \mathbb{V}(N)$. Fix $h \in \mathfrak{U} \setminus N$, set $M := N \cup \{h\}$, and let $w \in \mathbb{V}(M)$ be given by

$$w = \sum_{T \subseteq N : T \neq \emptyset} \lambda_T(v) \cdot u_T^M + \sum_{T \subseteq N \setminus \{i,j\}} \left[\lambda_{T \cup \{i\}}(v) \cdot u_{T \cup \{h\}}^M + \lambda_{T \cup \{i,j\}}(v) \cdot u_{T \cup \{i,h\}}^M + \lambda_{T \cup \{i,j\}}(v) \cdot u_{T \cup \{j,h\}}^M \right],$$

that is, a player h is added to v such that (**) i and j remain symmetric in w, (***) h is symmetric to both i and j in w, and (****) $w_{-h} = v$.

Since i and j are symmetric in w, they are second-order symmetric with respect to any $k \in M \setminus \{i, j\}$ in w. Hence, we have

$$\varphi_k(w_{-i}) \overset{\mathbf{2S}}{=} \varphi_k(w_{-j}) \qquad \text{for all } k \in M \setminus \{i, j\}. \tag{10}$$

Now, we obtain

$$\begin{aligned} \varphi_j(w_{-i}) &\overset{\mathbf{E}}{=} w_{-i}(M \setminus \{i\}) - \sum_{k \in N \setminus \{i,j\}} \varphi_k(w_{-i}) \\ &\overset{(**),(10)}{=} w_{-j}(M \setminus \{j\}) - \sum_{k \in N \setminus \{i,j\}} \varphi_k(w_{-j}) \\ &\overset{\mathbf{E}}{=} \varphi_i(w_{-j}). \end{aligned}$$

In view of (***), we analogously obtain

$$\varphi_j (w_{-h}) = \varphi_h (w_{-j}) \qquad \text{and} \qquad \varphi_i (w_{-h}) = \varphi_h (w_{-i}) . \tag{11}$$

Finally, we have

$$\varphi_i (v) \overset{(****)}{=} \varphi_i (w_{-h}) \overset{(11)}{=} \varphi_h (w_{-i}) \overset{(10)}{=} \varphi_h (w_{-j}) \overset{(11)}{=} \varphi_j (w_{-h}) \overset{(****)}{=} \varphi_j (v) ,$$

which concludes the proof. □

4. The Owen value

In this section, we first survey the characterization of the Owen value by Khmelnitskaya and Yanovskaya (2007). Then, we provide a second-order version of this characterization similar to the second-order characterization of the Shapley value by Casajus (2021) as surveyed in Section 3.

4.1. The (first-order) characterization by Khmelnitskaya and Yanovskaya (2007)

Khmelnitskaya and Yanovskaya (2007) generalize the characterization of the Shapley value due to Young (1985). This characterization indicates that the Owen value is the unique efficient CS solution that reflects both the players' and the components' (first-order) productivities in terms of the players' (first-order) payoffs.[8]

Efficiency, E. For all $N \in \mathcal{N}$ and $(v, \mathcal{P}) \in \mathbb{VP}(N)$, we have $\sum_{\ell \in N} \varphi_\ell (v, \mathcal{P}) = v(N)$.

Marginality, M. For all $N \in \mathcal{N}$, $(v, \mathcal{P}), (w, \mathcal{P}) \in \mathbb{VP}(N)$, and $i \in N$ such that $v(S \cup \{i\}) - v(S) = w(S \cup \{i\}) - w(S)$ for all $S \subseteq N \setminus \{i\}$, we have $\varphi_i (v, \mathcal{P}) = \varphi_i (w, \mathcal{P})$.

Symmetry within components, SwC. For all $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, $P \in \mathcal{P}$, and $i, j \in P$ such that i and j are symmetric in v, we have $\varphi_i (v) = \varphi_j (v)$.

For all $N \in \mathcal{N}$ and $(v, \mathcal{P}) \in \mathbb{VP}(N)$, the components $P, Q \in \mathcal{P}$ are called symmetric in $(v, \mathcal{P})$, if

$$v\left(P \cup \bigcup_{C \in \mathcal{C}} C\right) - v\left(\bigcup_{C \in \mathcal{C}} C\right) = v\left(Q \cup \bigcup_{C \in \mathcal{C}} C\right) - v\left(\bigcup_{C \in \mathcal{C}} C\right)$$

for all $\mathcal{C} \subseteq \mathcal{P} \setminus \{P, Q\}$, that is, if and only if the representatives of P and Q are symmetric in the intermediate game $v_{\mathcal{P}}$.

Symmetry across components, SaC. For all $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, and $P, Q \in \mathcal{P}$ such that P and Q are symmetric in $(v, \mathcal{P})$, we have $\varphi_P (v, \mathcal{P}) = \varphi_Q (v, \mathcal{P})$.

[8] Recently, Hu (2021, Theorem 3.2) kind of rediscovered this characterization. Instead of marginality, he uses coalitional strategic equivalence (Chun, 1989). Nowadays, however, it is well understood that coalitional strategic equivalence is equivalent to marginality (see, for example, Casajus, 2011, Footnote 3). Coalitional strategic equivalence: For all $N \in \mathcal{N}$, $T \subseteq N$, $T \neq \emptyset$, $i \in N \setminus T$, $\xi \in \mathbb{R}$, and $v \in \mathbb{V}(N)$, we have $\varphi_i (v) = \varphi_i \left(v + \xi \cdot u_T^N\right)$.

Efficiency and marginality are just the CS versions of the properties for (TU) solutions with the same name and with the same interpretation. Symmetry within components is a natural relaxation of symmetry within the CS framework. Symmetry across components treats the components as players: equally productive components should obtain the same payoff as expressed by the sum of their members' payoffs. Moreover, both symmetry within components and symmetry across component can be viewed as generalizations of symmetry. Whereas the former is equivalent to symmetry for the trivial coalition structure $\{N\}$, the latter is so for the atomistic coalition structure $\{\{i\} \mid i \in N\}$.

Theorem 5 (Khmelnitskaya and Yanovskaya, 2007). *The Owen value is the unique CS solution for $\mathbb{VP}$ that satisfies efficiency (**E**) symmetry within components (**SwC**), symmetry across components (**SaC**), and marginality (**M**).*

4.2. A second-order characterization

In this subsection, we simultaneously translate the second-order characterization of the Shapley value to CS solutions and the (first-order) characterization of the Owen value to the second-order framework.

Second-order marginality, 2M. For all $N \in \mathcal{N}$, $(v, \mathcal{P}), (w, \mathcal{P}) \in \mathbb{VP}(N)$, and $i, j \in N$, $i \neq j$ such that

$$\begin{aligned}&[v(T \cup \{i,j\}) - v(T \cup \{i\})] - [v(T \cup \{j\}) - v(T)] \\ &\quad = [w(T \cup \{i,j\}) - w(T \cup \{i\})] - [w(T \cup \{j\}) - w(T)]\end{aligned}$$

for all $T \subseteq N \setminus \{i,j\}$, we have

$$\varphi_j(v, \mathcal{P}) - \varphi_j(v_{-i}, \mathcal{P}_{-i}) = \varphi_j(w, \mathcal{P}) - \varphi_j(w_{-i}, \mathcal{P}_{-i}).$$

In essence, this property is just a restatement of second-order marginality for TU games, where the coalition structure is fixed but can be ignored otherwise. Therefore, the proof of Proposition 3 runs through smoothly within the framework of CS games and we obtain

Proposition 6. *If a solution for $\mathbb{VP}$ satisfies second-order marginality (**2M**), then it satisfies marginality (**M**).*

Second-order symmetry within components, 2SwC. For all $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, $P \in \mathcal{P}$, $i, j \in P$, and $k \in N \setminus P$ such that i and j are second-order symmetric with respect to k in v, we have

$$\varphi_k(v, \mathcal{P}) - \varphi_k(v_{-i}, \mathcal{P}_{-i}) = \varphi_k(v, \mathcal{P}) - \varphi_k(v_{-j}, \mathcal{P}_{-j}).$$

This property restricts second-order symmetry for TU games to players within the same component. Yet, the coalition structure can be ignored regarding the third player to whom the second-order marginal contributions and the second-order payoffs are related. The proof of Proposition 4 essentially runs through smoothly with second-order symmetry within components instead of symmetry within components and symmetry within components instead of symmetry: one just has to put player h into the component containing players i and j. Hence, we obtain

Proposition 7. *If a solution for $\mathbb{V}$ satisfies strong second-order symmetry within components (**2SwC**) and efficiency (**E**), then it satisfies symmetry within components (**SwC**).*

In order to obtain a second-order version of symmetry across components, we first provide the notion of second-order symmetry of components. For all $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, and $A, B, C \in \mathcal{P}$ pairwise different, components A and B are called **second-order symmetric with respect to component** C in $(v, \mathcal{P})$ if

$$\left[v\left(C \cup A \cup \bigcup_{D\in\mathcal{D}} D\right) - v\left(A \cup \bigcup_{D\in\mathcal{D}} D\right)\right] - \left[v\left(C \cup \bigcup_{D\in\mathcal{D}} D\right) - v\left(\bigcup_{D\in\mathcal{D}} D\right)\right]$$
$$= \left[v\left(C \cup B \cup \bigcup_{D\in\mathcal{D}} D\right) - v\left(B \cup \bigcup_{D\in\mathcal{D}} D\right)\right] - \left[v\left(C \cup \bigcup_{D\in\mathcal{D}} D\right) - v\left(\bigcup_{D\in\mathcal{D}} D\right)\right]$$

for all $\mathcal{D} \subseteq \mathcal{P} \setminus \{A, B, C\}$.

Remark 8. *Note that the components A and B are second-order symmetric with respect to component C in $(v, \mathcal{P})$ if and only if their representatives $\imath_A$ and $\imath_B$ are second-order symmetric with respect to the representative $\imath_C$ of component C in the intermediate game $v_{\mathcal{P}}$.*

Second-order symmetric components are equally second-order productive with respect to a third component. Therefore, if a CS solution is intended to reflect the components' second-order productivities in terms of their second-order payoffs, it seems to be plausible that the second-order payoffs of second-order symmetric components are the same.

Second-order symmetry across components, 2SaC. For all $N \in \mathcal{N}$, $(v, \mathcal{P}) \in \mathbb{VP}(N)$, and $A, B, C \in \mathcal{P}$ pairwise different such that A and B are second-order symmetric with respect to C in v, we have

$$\varphi_C(v, \mathcal{P}) - \varphi_C(v_{-A}, \mathcal{P}_{-A}) = \varphi_C(v, \mathcal{P}) - \varphi_C(v_{-B}, \mathcal{P}_{-B}).$$

Using the general idea of the proof of Proposition 4 one shows that second-order symmetry across components and efficiency imply symmetry across components.

Proposition 9. *If a solution for $\mathbb{VP}$ satisfies second-order symmetry across components (**2SaC**) and efficiency (**E**), then it satisfies symmetry across components (**SaC**).*

Proof. Let the CS solution φ satisfy **2SaC** and **E**. If $|\mathcal{P}| = 1$, then nothing is to show. Let now $N \in \mathcal{N}$ and $(v, \mathcal{P}) \in \mathbb{VP}(N)$ be such that $|\mathcal{P}| > 1$. Moreover, let (*) $P, Q \in \mathcal{P}$, $P \neq Q$ be symmetric in v. Fix $h \in \mathfrak{U} \setminus N$, set $M := N \cup \{h\}$ and $\mathcal{Q} := \mathcal{P} \cup \{\{h\}\}$, and let

$w \in \mathbb{V}(M)$ be given by

$$\begin{aligned} w = & \sum_{T \subseteq N : T \neq \emptyset} \lambda_T(v) \cdot u_T^M \\ & + \sum_{T \subseteq N \setminus (P \cup Q)} \sum_{S \subseteq P : S \neq \emptyset} \lambda_{T \cup S}(v) \cdot u_{T \cup \{h\}}^M \\ & + \sum_{T \subseteq N \setminus (P \cup Q)} \sum_{S \subseteq P \cup Q : S \cap P \neq \emptyset, S \cap Q \neq \emptyset} \lambda_{T \cup \{i,j\}}(v) \cdot u_{T \cup (S \setminus Q) \cup \{h\}}^M \\ & + \sum_{T \subseteq N \setminus (P \cup Q)} \sum_{S \subseteq P \cup Q : S \cap P \neq \emptyset, S \cap Q \neq \emptyset} \lambda_{T \cup \{i,j\}}(v) \cdot u_{T \cup (S \setminus P) \cup \{h\}}^M, \end{aligned}$$

that is, $\{h\}$ is added to $(v, \mathcal{P})$ such that (**) P and Q remain symmetric in $(w, \mathcal{Q})$, (***) $\{h\}$ is symmetric to both P and Q in w, and (****) $\left(w_{-\{h\}}, \mathcal{Q}_{-\{h\}}\right) = (v, \mathcal{P})$.

Since P and Q are symmetric in $(w, \mathcal{Q})$, they are second-order symmetric with respect to any $R \in \mathcal{Q} \setminus \{P, Q\}$ in $(w, \mathcal{Q})$. Hence, we have

$$\varphi_R(w_{-P}, \mathcal{Q}_{-P}) \overset{\mathbf{2SaC}}{=} \varphi_R(w_{-Q}, \mathcal{Q}_{-Q}) \qquad \text{for all } R \in \mathcal{Q} \setminus \{P, Q\}. \tag{12}$$

Now, we obtain

$$\begin{aligned} \varphi_Q(w_{-P}, \mathcal{Q}_{-P}) & \overset{\mathbf{E}}{=} w_{-P}(M \setminus P) - \sum_{R \in \mathcal{Q} \setminus \{P,Q\}} \varphi_R(w_{-P}, \mathcal{Q}_{-P}) \\ & \overset{(**),(12)}{=} w_{-Q}(M \setminus Q) - \sum_{R \in \mathcal{Q} \setminus \{P,Q\}} \varphi_R(w_{-Q}) \\ & \overset{\mathbf{E}}{=} \varphi_P(w_{-Q}, \mathcal{Q}_{-Q}). \end{aligned}$$

In view of (***), we analogously obtain

$$\varphi_Q\left(w_{-\{h\}}, \mathcal{Q}_{-\{h\}}\right) = \varphi_{\{h\}}(w_{-Q}, \mathcal{Q}_{-Q}) \tag{13}$$

and

$$\varphi_P\left(w_{-\{h\}}, \mathcal{Q}_{-\{h\}}\right) = \varphi_{\{h\}}(w_{-P}, \mathcal{Q}_{-P}). \tag{14}$$

Finally, we have

$$\begin{aligned} \varphi_P(v, \mathcal{P}) & \overset{(****)}{=} \varphi_P\left(w_{-h}, \mathcal{Q}_{-\{h\}}\right) \\ & \overset{(14)}{=} \varphi_{\{h\}}(w_{-P}, \mathcal{Q}_{-P}) \\ & \overset{(12)}{=} \varphi_{\{h\}}(w_{-Q}, \mathcal{Q}_{-Q}) \\ & \overset{(13)}{=} \varphi_Q\left(w_{-\{h\}}, \mathcal{Q}_{-\{h\}}\right) \overset{(****)}{=} \varphi_Q(v, \mathcal{P}), \end{aligned}$$

which concludes the proof. □

Propositions 6, 7, and 9, allow us to "transfer" Theorem 5 to the second-order framework. We obtain

Theorem 10. *The Owen value is the unique CS solution for $\mathbb{VP}$ that satisfies efficiency (**E**), second-order marginality (**2M**), second-order symmetry within components (**2SwC**), and second-order symmetry across components (**2SaC**).*

Proof. It is well-known that the Owen value satisfies **E**. Straightforward but tedious calculations using (5b) show the following formulas for the second-order Owen value payoffs in terms of second-order marginal contributions. Let $N \in \mathcal{N}$, $i, k \in N$, $i \neq k$, and $(v, \mathcal{P}) \in \mathbb{V}(N)$. If $k \in \mathcal{P}(i)$, then

$$\begin{aligned}
&\mathrm{Ow}_k(v,\mathcal{P}) - \mathrm{Ow}_k(v_{-i},\mathcal{P}_{-i}) \\
&= \sum_{\mathcal{C}\subseteq\mathcal{P}\setminus\{\mathcal{P}(k)\}} \sum_{S\subseteq\mathcal{P}(i)\setminus\{i,k\}} \left[\frac{v\left(S\cup\{i,k\}\cup\bigcup_{C\in\mathcal{C}} C\right) - v\left(S\cup\{i\}\cup\bigcup_{C\in\mathcal{C}} C\right)}{|\mathcal{P}(k)|\cdot\binom{|\mathcal{P}(k)|-1}{s+1}\cdot|\mathcal{P}|\cdot\binom{|\mathcal{P}|-1}{|\mathcal{C}|}} \cdots \right. \\
&\qquad \left. - \frac{v\left(S\cup\{k\}\cup\bigcup_{C\in\mathcal{C}} C\right) - v\left(S\cup\bigcup_{C\in\mathcal{C}} C\right)}{|\mathcal{P}(k)|\cdot\binom{|\mathcal{P}(k)|-1}{s+1}\cdot|\mathcal{P}|\cdot\binom{|\mathcal{P}|-1}{|\mathcal{C}|}} \right].
\end{aligned} \tag{15}$$

If $k \in N \setminus \mathcal{P}(i)$, then

$$\begin{aligned}
&\mathrm{Ow}_k(v,\mathcal{P}) - \mathrm{Ow}_k(v_{-i},\mathcal{P}_{-i}) \\
&= \sum_{\mathcal{C}\subseteq\mathcal{P}\setminus\{\mathcal{P}(k),\mathcal{P}(i)\}} \sum_{S\subseteq\mathcal{P}(k)\setminus\{k\}} \left[\frac{v\left(S\cup\{k\}\cup\mathcal{P}(i)\cup\bigcup_{C\in\mathcal{C}} C\right) - v\left(S\cup\mathcal{P}(i)\cup\bigcup_{C\in\mathcal{C}} C\right)}{|\mathcal{P}(k)|\cdot\binom{|\mathcal{P}(k)|-1}{s}\cdot|\mathcal{P}|\cdot\binom{|\mathcal{P}|-1}{|\mathcal{C}|+1}} \cdots \right. \\
&\qquad \left. - \frac{v\left(S\cup\{k\}\cup(\mathcal{P}(i)\setminus\{i\})\cup\bigcup_{C\in\mathcal{C}} C\right) - v\left(S\cup(\mathcal{P}(i)\setminus\{i\})\cup\bigcup_{C\in\mathcal{C}} C\right)}{|\mathcal{P}(k)|\cdot\binom{|\mathcal{P}(k)|-1}{s}\cdot|\mathcal{P}|\cdot\binom{|\mathcal{P}|-1}{|\mathcal{C}|+1}} \right].
\end{aligned} \tag{16}$$

From (15) and (16) it is immediate that the Owen value satisfies **2M** and **2SwC**. By Remark **??** and in view of the well-known fact that the Owen values satisfies **IG**, it also satisfies **2SaC**.

Let the CS solution φ satisfy **E**, **2M**, **2SwC**, and **2SaC**. By Propositions 6, 7, and 9, the CS solution φ satisfies **M**, **SwC**, and **SaC**. By Theorem 5, we have $\varphi = \mathrm{Ow}$. □

Remark 11. *The characterization in Theorem 10 is non-redundant for $|N| > 1$. The zero CS solution, Z, given by $\mathrm{Z}_i(v,\mathcal{P}) := 0$ for all $N \in \mathcal{N}$, $(v,\mathcal{P}) \in \mathbb{VP}(N)$, and $i \in N$ satisfies* *all properties but efficiency. The component egalitarian CS solution,* CE, *given by*

$$\mathrm{CE}_i(v,\mathcal{P}) := \frac{v(N)}{|\mathcal{P}(i)|\cdot|\mathcal{P}|}$$

for all $N\in\mathcal{N}$, $(v,\mathcal{P})\in\mathbb{VP}(N)$, and $i\in N$ satisfies all properties but marginality. Fix a bijection $\varrho:\mathfrak{U}\to\mathbb{N}$. For any $N\in\mathcal{N}$ and $\mathcal{P}\in\mathfrak{P}(N)$, let

$$\begin{aligned}&R(N,\mathcal{P},\varrho)\\ &:= \{\rho\in R(N,\mathcal{P}) \mid \text{for all } P\in\mathcal{P} \text{ and } i,j\in P:\rho(i)>\rho(j) \text{ if and only if } \varrho(i)>\varrho(j)\}.\end{aligned}$$

The ϱ-Owen value, Ow^{ϱ}, *given by*

$$\mathrm{Ow}_i^{\varrho}(v,\mathcal{P}) := \sum_{\rho\in R(N,\mathcal{P},\varrho)} MC_i^v(\rho)$$

for all $N\in\mathcal{N}$, $(v,\mathcal{P})\in\mathbb{VP}(N)$, and $i\in N$ satisfies all properties but second-order symmetry within components. The Shapley value for CS games ignoring the coalition structure satisfies all properties but second-order symmetry across components.

5. Concluding remarks

In this paper, we suggest a characterization of the Owen value indicating that the latter is the unique efficient CS solution that reflects the players' and components' second-order productivities in terms of their second-order payoffs. The natural question now arises whether this may hold true for higher-order productivities and higher-order payoffs. In view of the results of Casajus (2020, Appendix A), the Owen value should satisfy the corresponding higher-order properties, whereas not being the unique efficient CS solution to do so.

Winter (1989) generalizes the Owen value to games enriched with a level structure, that is, a finite sequence of coalition structures becoming successively finer. Khmelnitskaya and Yanovskaya (2007, Theorem 2) indicate how their characterization can be extended to this level structure value. We leave it to the reader to provide the obvious extension of our characterization of the Owen value to that level structure value.

6. Acknowledgements

We are grateful to a number of referees who commented on Casajus (2021) and thereby also improved the exposion of the ideas in this paper. André Casajus: Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 388390901.

References

Albizuri, M. J., 2008. Axiomatizations of the Owen value without efficiency. Mathematical Social Sciences 55, 78–89.

Alonso-Meijide, J. M., Casas-Méndez, B., González-Rueda, A. M., Lorenzo-Freire, S., 2014. Axiomatic of the Shapley value of a game with a priori unions. TOP 22, 749–770.

Alonso-Meijide, J. M., Fiestras-Janeiro, M. G., 2002. Modification of the Banzhaf value for games with a coalition structure. Annals of Operations Research 109, 213–227.

Aumann, R. J., Drèze, J. H., 1974. Cooperative games with coalition structures. International Journal of Game Theory 3, 217–237.

Calvo, E., Lasaga, J., Winter, E., 1996. The principle of balanced contributions and hierarchies of cooperation. Mathematical Social Sciences 31, 171–182.

Casajus, A., 2010. Another characterization of the Owen value without the additivity axiom. Theory and Decision 69 (4), 523–536.

Casajus, A., 2011. Differential marginality, van den Brink fairness, and the Shapley value. Theory and Decision 71 (2), 163–174.

Casajus, A., 2020. Second-order productivity, second-order payoffs, and the Shapley value. HHL Working Paper No. 187, HHL Leipzig Graduate School of Management, Leipzig, Germany.

Casajus, A., 2021. Second-order productivity, second-order payoffs, and the Shapley value. Discrete Applied Mathematics 304, 212–219.

Casajus, A., Huettner, F., 2018. Decomposition of solutions and the Shapley value. Games and Economic Behavior 13 (3), 1–23.

Chun, Y., 1989. A new axiomatization of the Shapley value. Games and Economic Behavior 1, 119–130.

Hamiache, G., 2001. The Owen value values friendship. International Journal of Game Theory 29, 517–532.

Harsanyi, J. C., 1959. A bargaining model for cooperative n-person games. In: Tucker, A. W., Luce, R. D. (Eds.), Contributions to the Theory of Games IV. Vol. 2. Princeton University Press, Princeton NJ, pp. 325–355.

Hart, S., Kurz, M., 1983. Endogenous formation of coalitions. Econometrica 51, 1047–1064.

Hu, X.-F., 2021. New axiomatizations of the Owen value. Mathematical Methods of Operations Research 93, 585–603.

Kamijo, Y., 2009. A two-step Shapley value for cooperative games with coalitionion structures. International Game Theory Review 1, 207–214.

Khmelnitskaya, A. B., Yanovskaya, E. B., 2007. Owen coalitional value without additivity axiom. Mathematical Methods of Operations Research 66 (2), 255–261.

Owen, G., 1977. Values of games with a priori unions. In: Henn, R., Moeschlin, O. (Eds.), Essays in Mathematical Economics & Game Theory. Springer, Berlin et al., pp. 76–88.

Owen, G., 1982. Modification of the Banzhaf-Coleman index for games with a priori unions. In: Holler, M. J. (Ed.), Power, Voting, and Voting Power. Physica-Verlag, Würzburg, Vienna, pp. 232–238.

Shapley, L. S., 1953. A value for n-person games. In: Kuhn, H., Tucker, A. (Eds.), Contributions to the Theory of Games. Vol. II. Princeton University Press, Princeton, pp. 307–317.

Winter, E., 1989. A value for cooperative games with levels structure of cooperation. International Journal of Game Theory 18, 227–240.

Young, H. P., 1985. Monotonic solutions of cooperative games. International Journal of Game Theory 14, 65–72.

LEIPZIG
GRADUATE SCHOOL
OF MANAGEMENT

HHL Leipzig Graduate School of Management
HHL is a university-level institution and ranks amongst the leading international business schools. The goal of the oldest business school in German-speaking Europe is to educate entrepreneurial, responsible and effective business leaders. HHL stands out for its excellent teaching, its clear research focus, its effective knowledge transfer into practice as well as its outstanding student services. Study programs include a Master in Management program, a Master with a specialization in Finance as well as an MBA program. All programs can be completed full-time or part-time. HHL also runs a Doctoral Program. The HHL Executive Education offers state-of-the-art management seminars for executive talents. Besides its main campus in Leipzig, the university maintains study locations in Cologne and Munich. HHL is accredited by AACSB International and ACQUIN. hhl.de The goal of the oldest business school in German-speaking Europe is to educate effective, responsible and entrepreneurially-minded leaders. HHL stands out for its excellent teaching, its clear research focus, its effective knowledge transfer into practice as well as its outstanding student services. According to the Financial Times, HHL ranks first in Germany and fifth globally for its entrepreneurship focus within the M.Sc. and EMBA programs. HHL is accredited by AACSB International. hhl.de

ISSN 1864-4562 (Online version)

For further HHL publications see **www.hhl.de/publications**